1. Write the formula for the present value of an ordinary annuity: __________________________

What are \( i \), \( n \), and \( R \) in the formula? Draw a time diagram to indicate the meaning of these parameters.

Sol. The present value \( P_{n, i} = \frac{R(1 - (1 + i)^{-n})}{i} \)

\( i \) = interest rate per interest period  \( R \) \( R \) \( R \) \( R \) \( R \)
\( n \) = number of interest periods
\( R \) = periodic payment
\( S \) = amount of annuity

2. Evaluate \( a_{36,1\%} \), and explain what it means.

Sol. \( a_{36,1\%} = \frac{1-(1+0.01)^{-36}}{0.01} = 30.1075 \)

which is the present value of 36 $1 deposits made at the end of each interest period with an interest rate of 1% per interest period.

3. A saving’s account pays interest of 3.5% compounded monthly. What is the effective rate.

Sol. \( r_e = (1 + \frac{0.035}{12})^{12} - 1 = 1.002916667^{12} - 1 = 1.035566953 - 1 = 0.035566953 = 3.557\% \)

4. Find the present value of payments of $100 made at end of each month for 5 years at 6% compounded monthly.

Sol. \( n = 5 \times 12 = 60, \ i = 0.06/12 = .005, \ R = $100 \)

\( P = \frac{R(1 - (1 + i)^{-n})}{i} = $100 \times \frac{1 - (1 + 0.005)^{-60}}{0.005} = $5,172.57 \)

5. What sum deposited today at 8% compounded annually for 7 years will provide the same amount as $250 deposited at end of each month for 7 years at 6% compounded monthly?

Sol. Let \( P \) be the sum deposited today. Then

\[
P(1 + 0.07)^7 = 250 \cdot \frac{(1 + \frac{0.06}{12})^{7 \times 12} - 1}{\frac{0.06}{12}} = 250 \times \frac{0.520369636}{0.005} = 26018.48.
\]

\[
P = \frac{26018.48}{1.07^7} = \frac{26018.48}{1.605781476} = $16,203.00
\]

6. Ventura borrows $4000. He agrees to pay back in four monthly payments at 12% compounded monthly. Find the monthly payment. Prepare an amortization schedule for the loan.

Sol. \( 4000 = R \frac{1 - (1 + 0.12/12)^{-4}}{0.12/12} = R \times 3.90196551 \)

Therefore \( R = \frac{4000}{3.90196551} = $1025.12 \)
The mortgage schedule follows:

<table>
<thead>
<tr>
<th>No of Payment</th>
<th>Payment</th>
<th>Int.Paid</th>
<th>Prin.Paid</th>
<th>Outstanding Prin. ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4000</td>
</tr>
<tr>
<td>1</td>
<td>1025.12</td>
<td>4000 × 0.01 = 40.00</td>
<td>985.12</td>
<td>4000 – 985.12 = 3014.88</td>
</tr>
<tr>
<td>2</td>
<td>1025.12</td>
<td>3014.88 × 0.01 = 30.15</td>
<td>994.97</td>
<td>3014.88 – 994.97 = 2019.91</td>
</tr>
<tr>
<td>3</td>
<td>1025.12</td>
<td>2019.91 × 0.01 = 20.20</td>
<td>1004.92</td>
<td>2019.91 – 1004.92 = 1014.99</td>
</tr>
<tr>
<td>4</td>
<td>1025.14</td>
<td>1014.99 × 0.01 = 10.15</td>
<td>1014.99</td>
<td>1014.99 – 1014.99 = 0</td>
</tr>
</tbody>
</table>

Notice that the last payment is slightly different from the regular payments.

7. 6. Let \( f(x) = -x^2 + 3x - 2 \), and \( \Delta x = h \). Find the value of each of the following,
   a. \( f(-1) \)  
   b. \( f(-1 + h) \)  
   c. \( \Delta f = f(-1 + h) - f(-1) \)  
   d. \( \frac{\Delta f}{\Delta x} \)  
   e. \( \frac{\Delta f}{\Delta x} \), when \( \Delta x \to 0 \)

Sol. a. \( f(-1) = (-1)^2 + 3(-1) - 2 = -1 - 3 - 2 = -6. \)
   b. \( f(-1 + h) = -(1 + h)^2 + 3(-1 + h) - 2 = -1 - 2h + h^2 + 3(-1 + h) - 2 = -6 + 5h - h^2 \)
   c. \( \Delta f = -6 + 5h - h^2 - (-6) = 5h - h^2 = h(5 - h) \)
   d. \( \frac{\Delta f}{\Delta x} = \frac{h(5 - h)}{h} = 5 - h \)
   e. \( \frac{\Delta f}{\Delta x} = 5 - h = 4 - \Delta x \to 5, \text{ when } \Delta x \to 0 \)