1. Write the formula for the future value of an ordinary annuity: 
   \[ S_{n,i} = R \left( \frac{(1+i)^n - 1}{i} \right) \]
   What are \( i \), \( n \), and \( R \) in the formula? Draw a time diagram to indicate the meaning of these parameters.

   Sol. The future value \( S_{n,i} = R \left( \frac{(1+i)^n - 1}{i} \right) \)

   - \( i \) = interest rate per interest period
   - \( n \) = number of interest periods
   - \( R \) = periodic payment
   - \( S \) = amount of annuity

2. Evaluate \( s_{48,0.02} \), and explain what it means.

   Sol. \( s_{48,0.02} = \frac{(1+0.02)^{48} - 1}{0.02} = 79.3535 \)

   which is the future value of 48 $1 deposits made at the end of each interest period with an interest rate of 2% per interest period.

3. A saving’s account pays interest of 3.8% compounded monthly. What is the effective rate.

   Sol. \( r_e = (1 + \frac{0.038}{12})^{12} - 1 = 1.003166667^{12} - 1 = 1.038668869 - 1 = 0.038668869 = 3.867\% \)

4. Find the present value of payments of $100 made at end of each month for 4 years at 6% compounded monthly.

   Sol. \( n = 4 \times 12 = 48, \ i = \frac{0.06}{12} = .005, \ R = 100 \)

   \[ P = R \left( \frac{1 - (1+i)^n}{i} \right) = 100 \times \frac{1 - (1 + 0.005)^{-48}}{0.005} = 4,258.03 \]

5. What sum deposited today at 7% compounded annually for 9 years will provide the same amount as $200 deposited at end of each month for 9 years at 6% compounded monthly?

   Sol. Let \( P \) be the sum deposited today. Then

   \[ P(1 + 0.07)^9 = \frac{200(1 + \frac{0.06}{12})^{9*12} - 1}{0.06} = 200 \times \frac{0.713699498}{0.005} = 28547.98. \]

   \[ P = \frac{28547.98}{1.07^9} = \frac{28547.98}{1.8384592} = 15,528.21 \]

6. Ventura borrows $5000. He agrees to pay back in four monthly payments at 12% compounded monthly. Find the monthly payment. Prepare an amortization schedule for the loan.

   Sol. \( 5000 = R \left( \frac{1 - (1 + \frac{0.12}{12})^{-4}}{0.12/12} \right) \rightarrow R \times 3.901965551 \) Therefore \( R = \frac{5000}{3.901965551} = 1281.41 \)
The mortgage schedule follows:

<table>
<thead>
<tr>
<th>No of Payment</th>
<th>Payment</th>
<th>Int.Paid</th>
<th>Prin.Paid</th>
<th>Outstanding Prin.($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5000</td>
</tr>
<tr>
<td>1</td>
<td>1281.41</td>
<td>5000 × 0.01 = 50.00</td>
<td>1231.41</td>
<td>5000 - 1231.41 = 3768.59</td>
</tr>
<tr>
<td>2</td>
<td>1281.41</td>
<td>3768.59 × 0.01 = 37.69</td>
<td>1243.72</td>
<td>3768.59 - 1243.72 = 2524.87</td>
</tr>
<tr>
<td>3</td>
<td>1281.41</td>
<td>2524.87 × 0.01 = 25.25</td>
<td>1256.16</td>
<td>2524.87 - 1256.16 = 1268.71</td>
</tr>
<tr>
<td>4</td>
<td>1281.40</td>
<td>1268.71 × 0.01 = 12.69</td>
<td>1260.71</td>
<td>1268.71 - 1260.71 = 0</td>
</tr>
</tbody>
</table>

Notice that the last payment is slightly different from the regular payments.

7. 6. Let \( f(x) = -x^2 + 2x - 3 \), and \( \Delta x = h \). Find the value of each of the following,

a. \( f(-1) \)  
b. \( f(-1 + h) \)  
c. \( \Delta f = f(-1 + h) - f(-1) \)  
d. \( \frac{\Delta f}{\Delta x} \)

e. \( \frac{\Delta f}{\Delta x} \), when \( \Delta x \to 0 \)

Sol. a. \( f(-1) = -(-1)^2 + 2(-1) - 3 = -1 - 2 - 3 = -6 \).

b. \( f(-1 + h) = -(-1 + h)^2 + 2(-1 + h) - 3 = -(1 - 2h + h^2) + 2(-1 + h) - 3 = -6 + 4h - h^2 \)

c. \( \Delta f = -6 + 4h - h^2 - (-6) = 4h - h^2 = h(4 - h) \)

d. \( \frac{\Delta f}{\Delta x} = \frac{h(4 - h)}{h} = 4 - h \)  
e. \( \frac{\Delta f}{\Delta x} = 4 - h = 4 - \Delta x \to 4, \text{when} \ \Delta x \to 0 \)