Mathematics and CS Club

Lessons Learned From Using Ranked Choice Voting in the 2021 NYC Democratic Primary and Election

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In November New York City voters elected Eric Adams Mayor for the City of New York.

Both the Democratic Party and the Republican party ran candidates as did some other parties.

Some of these minor (third) parties selected the same candidate to run as someone on the Democratic or Republican party ticket.

Historically, the person who runs on the Democratic ticket in New York City for mayor usually will become mayor!

(8 names appeared on the ballot in November, 2021!)

In practice the person who wins the Democratic primary election to be the candidate for mayor, often becomes the mayor of New York City.

(Eric Adams won the Democratic primary.)

What was unusual about the most recent primary for the Democratic Party was that it was decided using "ranked choice voting," a *major departure* from plurality voting.

Is "ranked choice voting" better than plurality, or just different? What will be the consequences for NYC politics of the regular use of ranked choice voting?

Can mathematics and computer science help with answering these questions?

What insights does mathematics offer into elections and voting?

To help understand elections let us look at a mathematical representation, known as a mathematical model, for an election.

A model or mathematical model is a simplified look using mathematics at the complex "reality" of some situation/problem in the world outside of mathematics.

A doll is a model for children of a real human being.

A model plane, train, or car is a simplified version of a real plane, train or car.

A model can sometimes be physical but often it is build using digital technologies, equations, or diagrams (graph theory- dots and lines diagram).

Components of an election or voting situation?

Before looking at this question in more detail, perhaps a few remarks about democracy, fairness, and elections are worthwhile.

We seem to be at a crossroad around the world where many countries with democratic traditions are straying from these traditions, becoming more authoritarian, and nationalistic.

On January 6, 2021, for example there was an effort to "overturn" the results of the 2020 election where Joseph Biden defeated Donald Trump in the election for President. Nothing like this had happened before in American history!!! The transition of "power" is one of the essential features of a democracy.

There were elections in Nazi Germany and in the Soviet Union but Hitler and Stalin once they initially came to power were not elected "democratically."

Aristotle wrote extensively on fairness.

He pointed out that one does not have to treat people exactly alike to be fair!

John Rawls (1921-2002)

Justice as Fairness

Other important thinkers here are

Jeremy Bentham; David Hume; John Locke; Adam Smith; Karl Marx; John Stuart Mills, Immanuel Kant

The economic system used in a country is not the same as the political system. America has a history of "*demonizing*" *leftist* but democratic governments.

Right wing despots have often been endorsed by America.

Utilitarianism (Bentham) - Most good for the most people. But how can one measure and compare people's pain or happiness?

Alternatively, make the least well off as well off as possible. (Rawls)

Musk, Gates, Zuckerberg and Bezos distort data which is measured in terms of "total" well being.

This is why often income is reported in terms of the median rather than the arithmetic mean.

Components in an election:
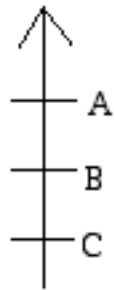
a. Candidates or choices

b. Voters or selectors

c. Ballot (a way for the voters to express information about the candidates (choices)

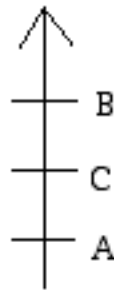d. Procedure for counting the ballots to select the winner(s)

(Sometimes called the "election method.)

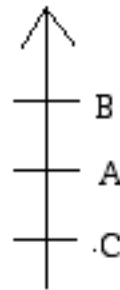Note: one can think of this as a function f mapping elections to a set of winners;

f(

A	B	B	
B	C	A	
C	A	.C	
499	498	3	votes

) to (winning set) (or ranking)

e. Goals for conducting the election

Comment:

* Will individuals or groups of voters complete their ballots honestly/sincerely or vote "strategically" to improve their "outcome?"

Comment:

Sometimes voters (or groups of voters) look beyond the merits of a particular candidate to whether this candidate's "divisiveness" might lead them to vote for someone other than their "favorite" candidate.
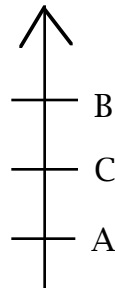
i. ordinary ballot (plurality ballots)
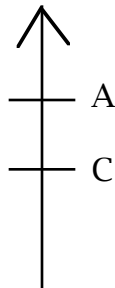
Vote for your favorite choice.

ii. preference, ordinal, or ranked ballot

List choices in preference order without ties. Sometimes one is limited as to the total number of candidates one can rank (five for NYC voting)
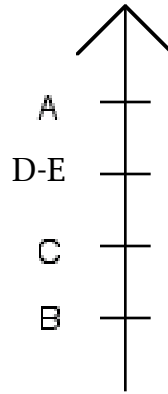
Three candidates all ranked without ties:

↑
— B
— C
— A

3-candidates but voter ranked only two candidates (called truncation)

↑
— A
— C

Five candidates ranked with ties; no truncation

A

D-E

C

B

A>D=E>C>B

Approval ballot:

Vote for those candidates you are willing to have serve.

Promoted recently by Steven Brams (political scientist) and Peter Fishburn (who died a few years ago; spent most of his career at Bell Laboratories)

Yes-No ballot
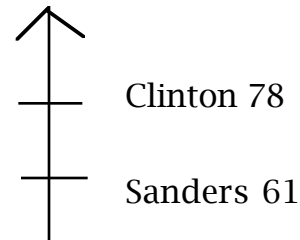
For each of the candidates/choices vote yes or no.

Note these different ballots require different "*skills*" of the person(s) who do the choosing.

Cardinal ballots

These are ballots which list an intensity of feeling, a mark (grade), or score for each candidate.

Same preference but intensity given:
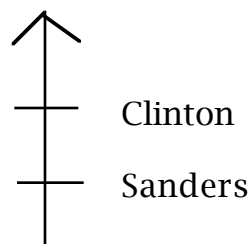
But what SCALE should one use to measure the "intensity" of preference, judgment, score (when judges give scores to ice skaters in competitions)

a. verbal

poor fair neutral good excellent

many variants (translate to French?)

Letter "grades"

A+ A A- B+ B- C+ C C- D+ D D- F

b. Numerical (Is a higher number stronger or weaker candidate?)

0 to 99; 0-100; 1-99 0-99

1-10; 0 -10

-3 to + 3; -5 to +5; -10 to + 10

d. Assumption about how much skill/knowledge the voters bring to expressing

e. How easy is it for groups of voters to take concerted action to result in a favorable outcome for their group?

f. A method of counting ballots to select a winner.

Comment: Some elections select a group of winners (e.g. committee elections)

Some well studied election decision methods using ranked (cardinal ballots) ballots:

a. Plurality
b. Run-off (based on first place vote)
c. Sequential run-off (IRV)
d. Borda Count
e. Condorcet
f. Baldwin (Borda Count run-off)
g. Coombs (Run-off using last place votes)

Some background related to these issues.

An election with 1000 voters and who voted using 3 of the six possible ballots with 3 candidates:

A	B	B
B	C	A
C	A	.C
499	498	3
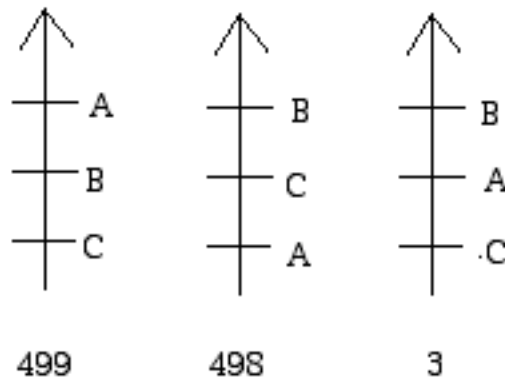
A>B>C is another way of "coding" the votes of the first 499 voters.

In this notation a tie is indicated by an = sign:

A>B=C>D

A>B=C=D

A voter who had no knowledge of B, C, and D might vote this way.

All voters here ranked all 3 candidates, but perhaps the candidate B might be unknown to some votes and would have preferred to be able to truncate their ballot if possible because, sincerely, they could not complete a ranking using all three choices.

A

C
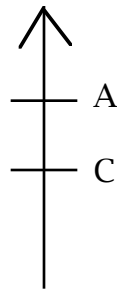
If the voting rules require strict ranking for all three candidates presumably candidates unfamiliar to the voter would appear towards the bottom.

Rather surprisingly perhaps mathematicians have also systematically investigated many aspects of fairness.

One notable example is that Lewis Carroll the author of *Alice in Wonderland* but less famously being a professor of mathematics in England was an early contributor to the mathematical theory of elections.

Mathematics has looked at fairness issues related to:

a. Elections and voting

b. Fair legislative representation

c. Weighted voting

d. Cost sharing

e. Fair allocation (fair division - cake cutting)

f. School choice (market design)

g. Fair vaccination programs

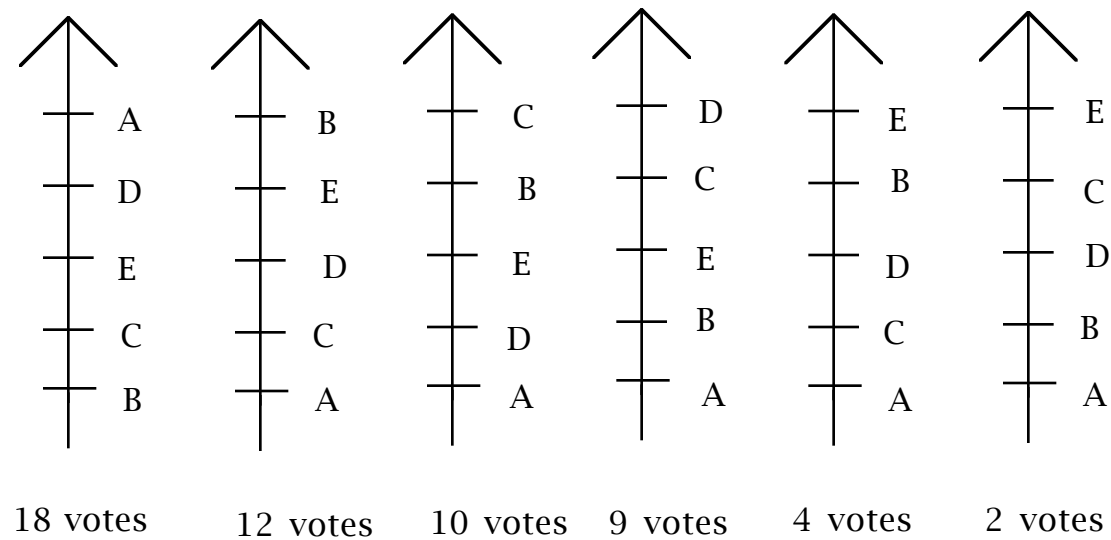i. Gerrymandering

j. College admissions

Who has the best claim to win the following election?

Higher preferences towards the top:

18 votes	12 votes	10 votes	9 votes	4 votes	2 votes
A	B	C	D	E	E
D	E	B	C	B	C
E	D	E	E	D	D
C	C	D	B	C	B
B	A	A	A	A	A

Note that in most American elections one merely votes for one's favorite choice.

However using these ranked or ordinal ballots one can get more nuanced views about the choices/candidates from the voters.

One might, instead of asking for RANKED ballots, ask the voters to assign points to each of the candidates rather than rank them -cardinal ballots.

Numbers can be used to count and numbers can be used to measure.

ordinal numbers (natural numbers)

cardinal numbers (real numbers)

Some appealing methods to conduct elections:

1. Plurality (winner gets the largest number of first place votes)

2. Run-off (If no candidate has a majority, eliminate all but the top two vote getters and hold an election between them

3. Sequential run-off (IRV - instant run-off voting) (If no candidate has a majority, eliminate the candidate with the lowest number of first place votes; transfer these votes to the other remaining candidate. Repeat until there is a single winner.

(To be used in NYC starting in 2021. Ballots will allow up to 5 choices.)

4. (Condorcet) Winner is the candidate, if there is one, who can beat all the other candidates in a two-way race.

5. (Borda Count) Given a ballet assign point to each candidate on the ballot in terms of how many candidates are below a given candidate on that ballot.

Example:

```
              ^
              |
        A    -+-
             |
       D-E   -+-
             |
        C    -+-
             |
        B    -+-
             |
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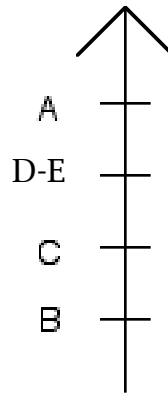
A gets 4 points
B gets 0 points
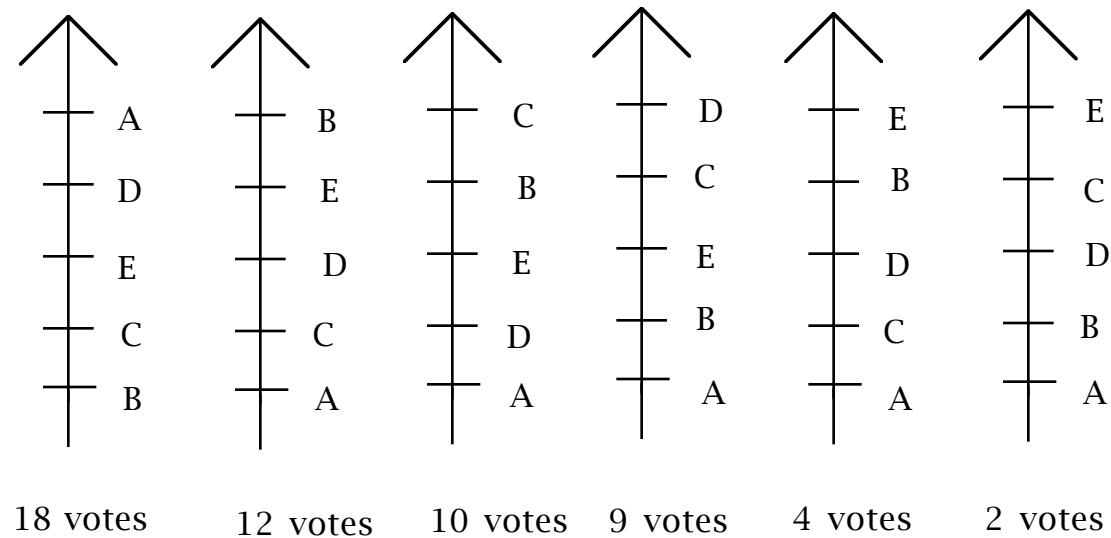C gets 1 point
D gets two point
E gets two points

If 10 voters with this ballot, multiply by 10.

Another notation: A > D=E > C > B

| 18 votes	12 votes	10 votes	9 votes	4 votes	2 votes
A	B	C	D	E	E
D	E	B	C	B	C
E	D	E	E	D	D
C	C	D	B	C	B
B	A	A	A	A	A

Perhaps surprisingly the 5 different methods just described give 5 different winners!

When a person wins an election perhaps it is less the "will of the people" rather than the method chosen to count the votes that matters!

Consequences of no Condorcet winner:

25	40	35	Votes
A	B	C	
B	C	A	
C	A	B	
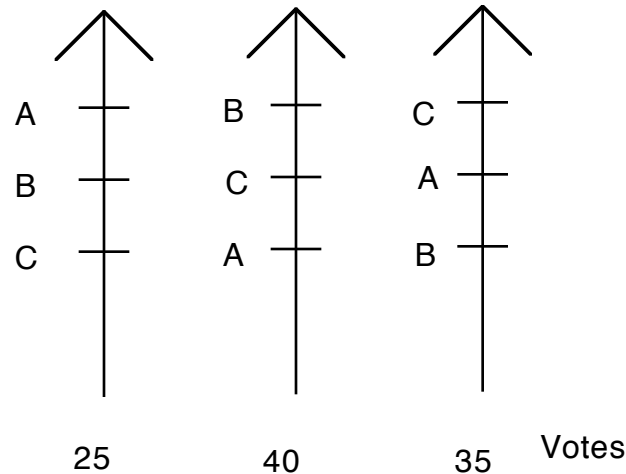
Note what happens if voting on items takes place sequentially in pairwise votes. Many real world legislatures work this way.

A	B	C	
B	C	A	
C	A	B	
25	40	35	Votes
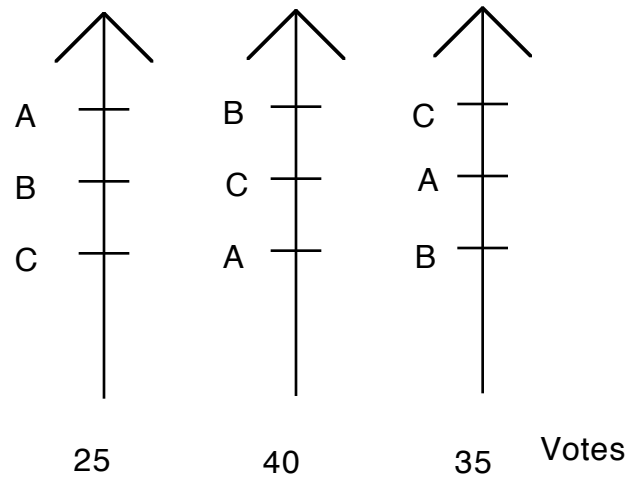
a. Vote on A vs. B; pit winner against C

A wins initially; C wins A vs C - C becomes law.

b. Vote on B vs. C; pit winner against A

B wins initially; A wins A vs B - A become law.

c. Vote on A vs. C; pit winner against B

C wins initially; B wins B vs. C - B becomes law

So how can one choose between different appealing methods?

Kenneth Arrow (City College graduate before CUNY existed) and winner of the Nobel Memorial Prize in Economics suggested the idea of seeing which nice FAIRNESS properties different methods obeyed and picking that method which obeyed the fairness rules one felt were important.

Examples of fairness rules:

1. Non-dictatorial

2. Non-imposed

3. Universal

4. Monotonic

5. Independence of irrelevant alternatives.

Arrow's Theorem:

There is no election method when one chooses among 3 or more candidates candidates using ranked ballots with ties allowed which obeys this list of fairness rules!

The mathematical model that Arrow builds involves:

a. Voters
b. Choices candidates
c. Ballot
d. Election decision method (a function mapping any election (individual choices) to a ranking (society choice).

More troubling and more general result.

When conducting elections with ordinal or point ballots (give each candidate some number of points from 0 to 99), does it ever help to misrepresent or "lie" about one's true feelings to help a particular candidate?

Voting of this kind is called *strategic*.

Satterthwaite-Gibbard Theorem:

When there are three or more candidates the only election decision method that cannot be manipulated is dictatorship!!

Sometimes well meaning distortions of the electoral process by polling companies, newspapers and television (broadcast and cable)

Polls

It seems to a characteristic of people that they like close races, whether it is a horse race or who will win an important election.

Thus, when there is a big gap between the way candidates poll this may set in motion activities to narrow the gap to make the race "more exciting." However, in politics ideally voters will not follow the lead of particular people in how they vote or vote the way they do because the Wall Street Journal or Fox News told them the way they vote.

(Perhaps people watch more news when an election races is close?)

Election night reporting:

For national elections the polls close in east coast states earlier than they close on the West Coast, not to mention the complication of Hawaii which is in a very different time zone to the rest of the country.

With ranked choice voting, and in elections where there are significant numbers of absentee ballots the official decision as to who will win the election typically takes considerable time after the polls close.

Thanks for listening!

Questions? Comments?

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