Content Matters

Moderator:

Joseph Malkevitch
York College (CUNY)

email:

malkevitch@york.cuny.edu

web page:

http://york.cuny.edu/~malk

K-12 Mathematics Education is in trouble:

but is the "problem" mainly:

the culture

the students

the teachers

the content

the system ?

Content matters

but what is best for different students might be different:

* content for STEM needs

* content for the needs of all students

* content for the workplace

* content for those "good" at mathematics

* content for those who have "trouble" with mathematics

etc.

Goals of all content of mathematics should include:

* building ambassadors for mathematics

* having future parents who are sophisticated in their views about mathematics and its importance

* exposing students to a variety of mathematical tools for solving problems of the workplace and daily life

* making students aware of domains where mathematics is applicable

* making students aware of careers that use mathematics

Important considerations:

* Mathematical Thinking

* Reasoning Skills and Problem Solving

* Mathematics Applied

* Applicable Mathematics

* Technology

Historically we have been very concerned with mathematical technique

Techniques:

0. Arithmetic
1. Geometry
2. Algebra
3. Trigonometry
4. Calculus (Single Variable and Multivariate)
5. Differential Equations
6. Linear (Matrix) Algebra
7. Modern Algebra
8. Probability and Statistics
9. Real Variables
10. Complex Variable
11. Combinatorics
12. Graph Theory
13. Coding Theory
14. Knot Theory
15. Partial Differential Equations

(Many more!)

Technique:

"older"

a. Solving linear equations

b. Solving quadratic equations

c. Arithmetic of complex numbers

"newer"

d. Matrix addition and multiplication

e. Find measures of central tendency

f. Computing probabilities

g. Counting permutations and combinations

Themes:

1. Optimization
2. Growth and Change
3. Information
4. Fairness and Equity
5. Risk
6. Shape and Space
7. Pattern and Symmetry
8. Order and Disorder
9. Reconstruction (from partial information)
10. Conflict and Cooperation
11. Unintuitive behavior

(Many more!)

Global Example:

Cell phones are a gift (curse?) of 20th century mathematics to humanity!

* **error correction codes and technology**

(seminal work of Richard Hamming)

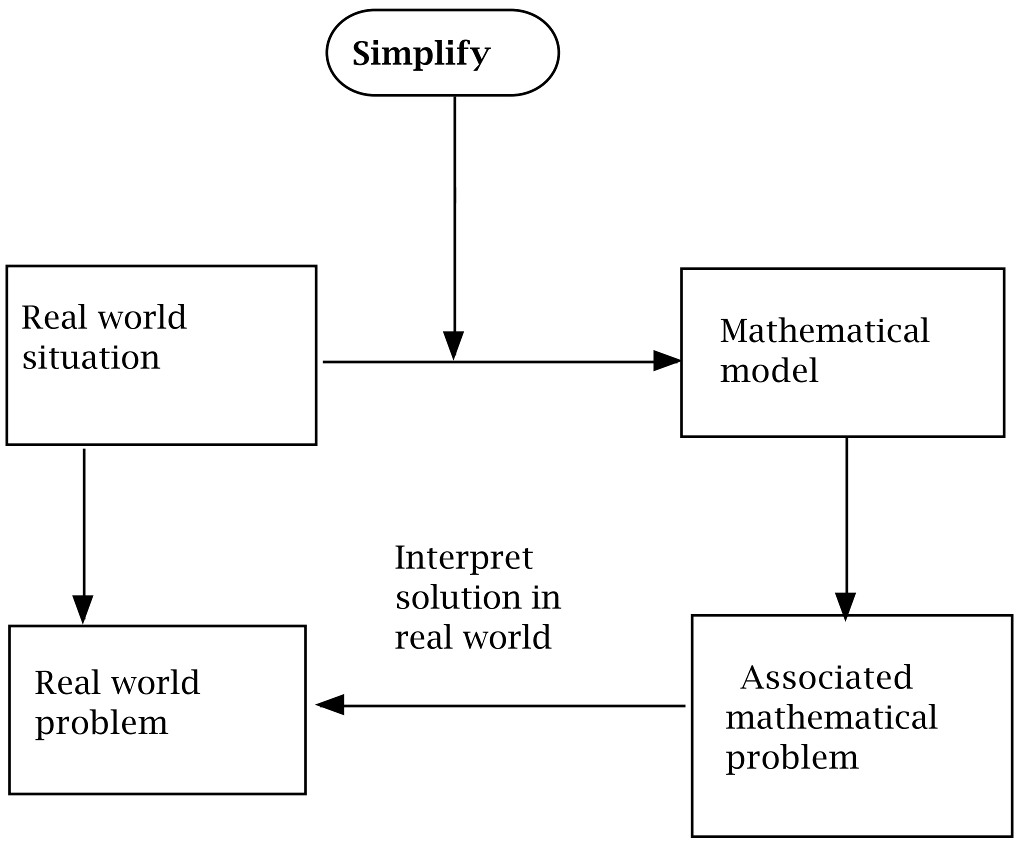
* **data compression technology**

(seminal work of David Huffman)

* **global positioning systems**

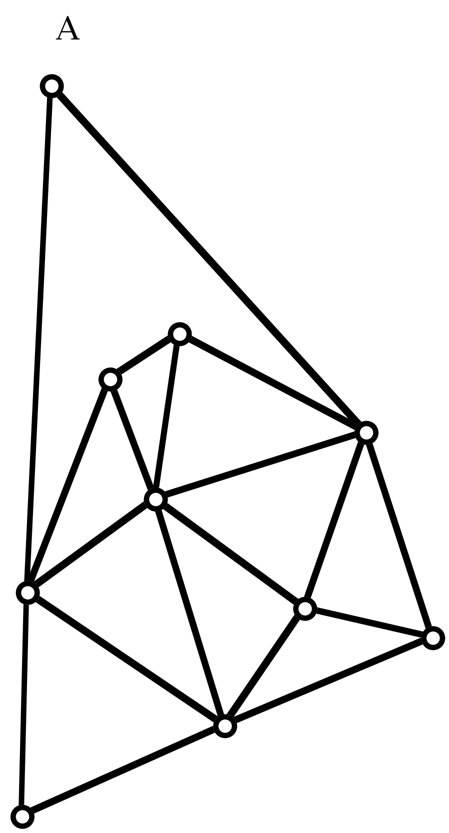
* **mathematics for frequency assignment of calls**

We should be teaching themes and techniques in a modeling framework throughout K -12!



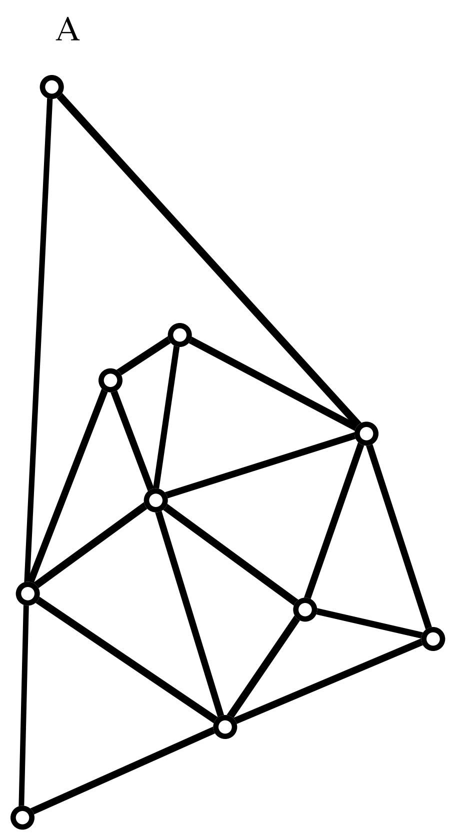
Example:

Is it possible to find an efficient pot hole inspection route, which starts and ends at A for the street network below?



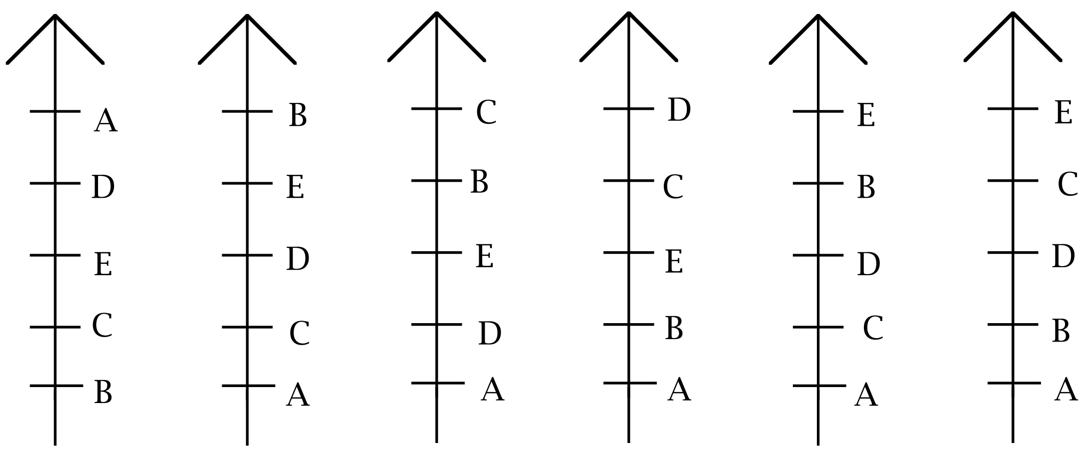
Example:

Is it possible to find an efficient route, which starts and ends at A and which traverses each of the "bridges" (represented by line segments) in the diagram below below?



Example:

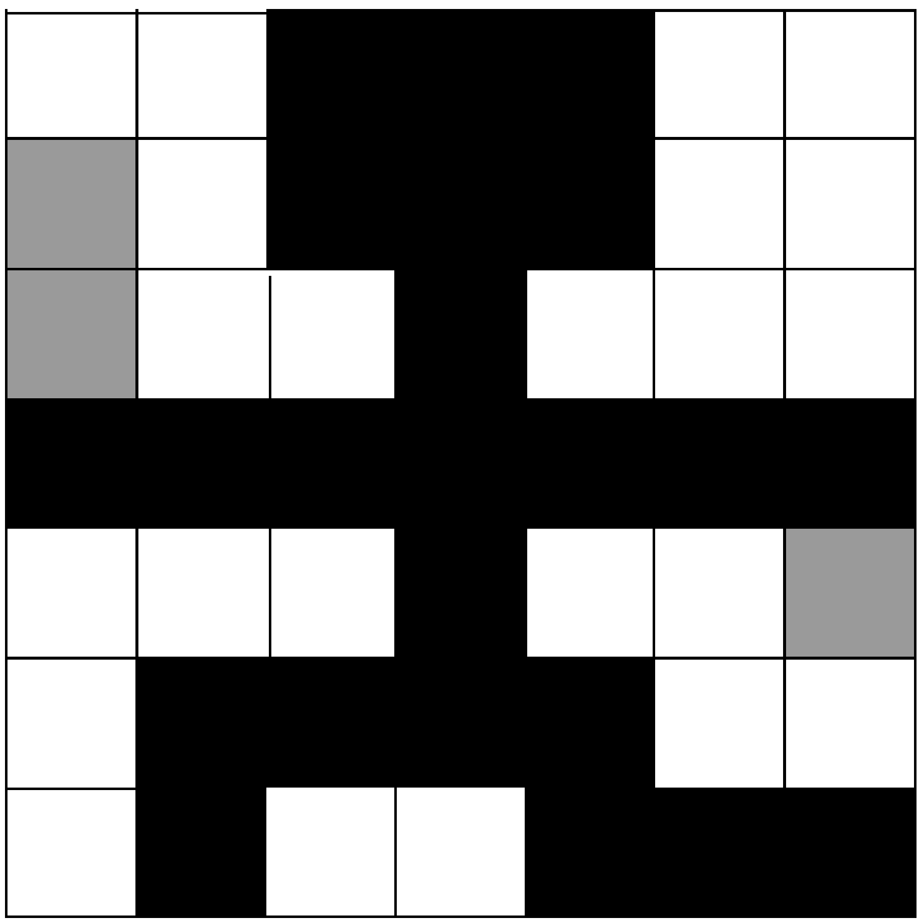
Who is the most "deserving" winner of this election and why?



Votes: 18 12 10 9 4 2

Example:

How would you send this image over a phone connection?



Example:

Is this a fair game?

Column

		C_1	C_2
Row	R_1	5	-2
	R_2	-10	4

Spell checker?

How does a spell checker work?

Idea:

Distance between two strings of the same length is the number of positions they differ in:

100011
001101

Hamming distance is 4

If you enter bneak into a word processor,

bneak is not a word in its dictionary

The world processor suggests words which are Hamming distance 1 from bneak under the assumption that you made one "typo."

bleak
break
sneak

Ideas like this are what made it possible to find distances between DNA strands and the songs of different birds, and the faces of different people!

Example:

A bankrupt company with with assets of $180 million must pay off two claimants with verifiable claims of:

A = $300 million

B = $200 million

What is a fair way for a Judge to settle the claims?

What fairness principle should one use?

* equality of what the players are given?

* equality of what the players lose?

* proportionality to claims?

* return fixed percentages on each claimed dollar?

etc.

Methods of solving such problems lead to many interesting methods and ideas:

* Maimonides method

* Contested garment rule

* Constrained optimization

* Shapely value

Principle:

Equalize loss:

If A given a

and B given b, then:

$a + b = 180$

$300 - a = 200 - b$

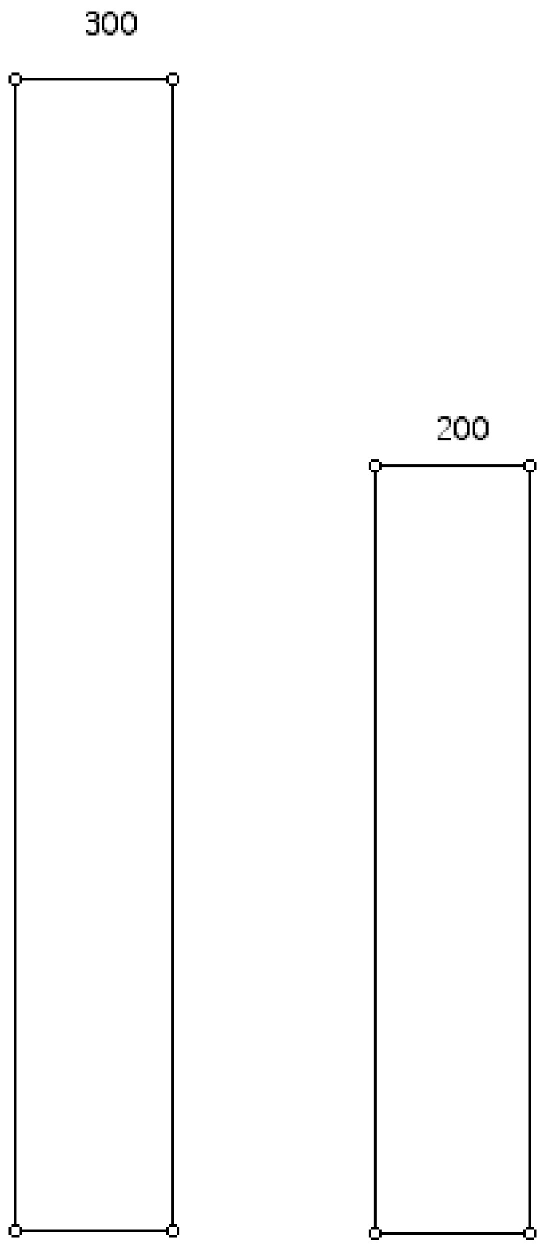
So solve:

$a + b = 180$
$-a + b = -100$

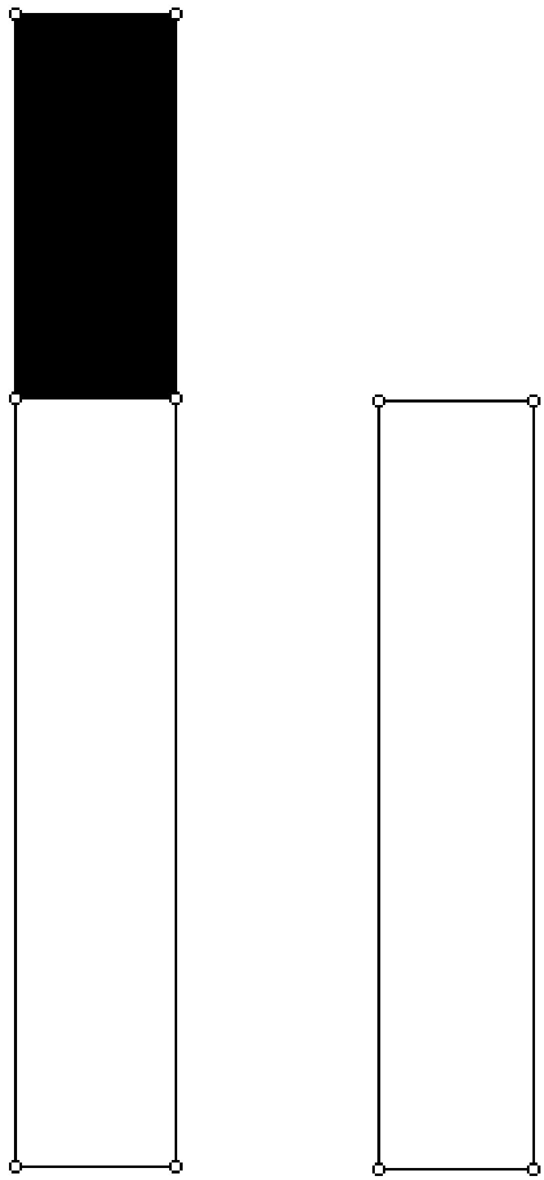
$b = 40$ and, hence, $a = 140$.

Geometric Solution:

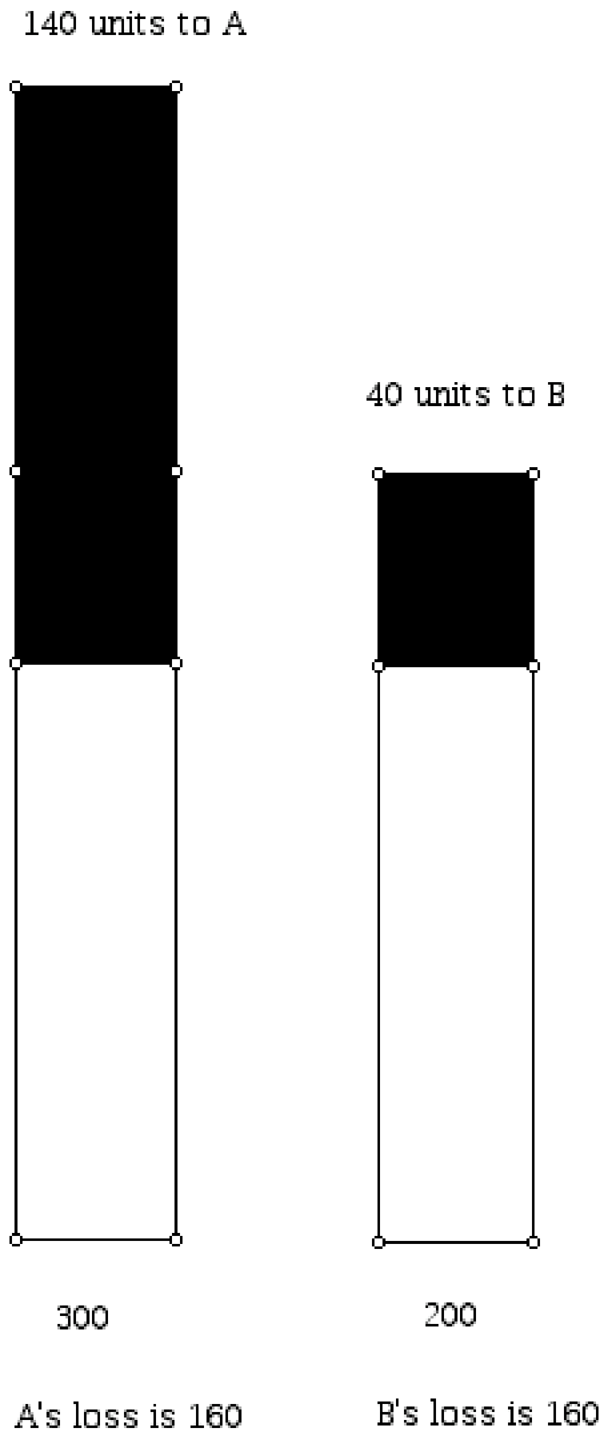
Diagram showing the losses for A and B before any money is returned:



We must first give A enough back so that the loss for A and B are the same!



Now A and B have equal losses but there is only 80 left to distribute. So we give 40 units to each of A and B:


140 units to A
40 units to B
300
200
A's loss is 160
B's loss is 160

We do not stop at 1900 in teaching history in our schools because there is so much important history prior to 1900!

We should not stop teaching mathematics developed after 1900 because there is so much important mathematics developed prior to 1900!

Content for teachers:

* high school mathematics teachers should major in mathematics

* you can not teach about topics you know nothing about

* teachers must know mathematics beyond what they actually teach

Content matters!

Students must develop reasoning skills, algorithmic skills, and modeling skills throughout K -12!